"Modal violations" in seismic engineering

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ABSTRACT

With increasing seismic hazard as advised by the NSHM 2022 for all of New Zealand, there is demand for the retrofit of our existing building stock to achieve higher earthquake resilience. One of the ways to achieve a cost-effective retrofit achieving higher resilience is by adding "viscous dampers" into the existing structure. Although the viscous phenomenon is well understood, and the technology has been around for more than 50 years, the induced mechanics created by these devices and their effect on the overall "structural dynamics" is still not well understood judging by the state-of-the-art industry practice. This paper sheds some light on the complex mechanics referred to here as "damper-structure-interaction". The paper, with the help of numerical examples, illustrates why the use of classical popular modal methods to understand such systems is flawed and why to do should be considered a "modal violation". The paper also illustrates that nonlinear time history analysis is the only analytical tool that should be used to verify the design of such systems.

1 INTRODUCTION

A conventional seismic design strategy relies on the structure absorbing seismic energy in a major earthquake by it enduring large inelastic deformations in a ductile fashion. The inelastic deformation results in heavy economic losses in the form of material damage to structural and non-structural elements. Recent earthquakes, Christchurch 2011, Tohoku 2011, Kaikoura 2016, Kumamoto 2016, all have resulted in heavy damage losses. For example, the loss incurred in the Christchurch earthquake was greater than NZD $45 billion which is about 20% of New Zealand's Gross Domestic Product. For the development of a sustainable human society, these types of losses cannot be accepted and should be minimised. This calls for alternative thinking in the way we design and build our infrastructure. The concept of seismic resilience that achieves our sustainability goals needs to be embedded in the design process.

One way to achieve this might be to retrofit existing structures with fluid viscous dampers. These provide actions which are out of phase with the usual structural actions during an earthquake. Properly specified, they can minimise inter-storey drift, absolute floor accelerations and foundation loading. Consequently, damage and loss of use can be reduced considerably – even in a major seismic event, significantly larger than the design event. Additionally, they require much less foundation intervention compared to other retrofitting technologies such as base-isolation where that may also be considered as an option.

However, the design of a structure that is to include fluid viscous dampers is not straightforward as the dampers interact with the structure (particularly when it is responding inelastically) in a complex dynamic way because of their dependence on velocity.

A number of simplified techniques have been developed over the years that treat the device as a viscous strut. We have investigated the validity of such methods by using mathematically-rigorous, physically-consistent nonclassical modal analysis. It is shown that these simplified techniques violate some of the fundamental mechanics applicable to a viscously-damped structure and hence result in solutions which may be incorrect.

We begin the paper describing the unique "damper-structure interaction phenomenon with a physics-based explanation. Next, we review the existing pseudo-static methods conceptually and then present a numerical study which illustrates the physical inconsistency in the so-called pseudo methods when applied to controlled structures using additional damping devices. A discussion of the ramifications of our findings and our conclusions follow.

2 DAMPER-STRUCTURE INTERACTION

This section mainly addresses the question, what happens when discrete viscous dampers get added to the structure? The simple answer to this is that the addition of discrete viscous dampers makes the damping of the structure non-proportional. *Now the pressing question is what do we really mean by non-proportional damping?*

This section attempts to provide a simplified theoretical explanation of the concept of non-proportionality using an elastic structure. It is based on published research, and describes the novel mechanics exhibited by structures incorporating these devices i. It is an elucidated summary of the following references: Chopra (2017), Clough & Penzien (1993), Veletsos & Ventura (1986), Hurty & Rubinstein (1964).

The governing equation of motion for a linear, dynamic, multi-degree-of-freedom structure subjected to a generic force vector varying with time is given as:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = f(t) \tag{1}$$

Here, M, C and K are the mass, damping and stiffness matrices, respectively. $f(t)$ is the force vector varying with time and, in the case of earthquake loading, this becomes $-MR\ddot{u}_g(t)$ where $\ddot{u}_g(t)$ is the ground-motion acceleration record, R is the directionality influence vector and (t) indicates variation of the quantity with time. $\ddot{u}, \dot{u}$ and u represent the relative accelerations, velocities, and displacements, respectively.

For a linearly elastic structure classical modal analysis is one of the most efficient methods for solving equation (1). The method works by identifying undamped modes or *classical normal modes or natural modes or principal modes* of free vibration which can simultaneously orthogonalize mass and stiffness matrices which results in the de-coupling of the equations of motion. *Physically, a natural mode is a deformed state of the structure where each degree of freedom in the structure executes a harmonic motion about a position of static* *equilibrium and where each of these degrees of freedom passes through its equilibrium position at the same instant and reaches its extremum at the same instant.*

Most building codes and standards assume that a structure possesses classical normal modes and by accepting classical normal mode methods of analysis assume that the structure is proportionally damped. The phenomenon of de-coupling results in the disaggregation of a complex multi-degree-of-freedom (MDOF) structure into a set of single-degree-of-freedom (SDOF) systems and the responses of the complex MDOF structure can be represented as the summation of the responses of the SDOFs.

Strictly speaking, classical modal methods are not applicable in seismic engineering if the structure is non-linear and even less applicable for structures with added supplemental damping as both conditions result in the structure being non-proportionally damped. In other words, the above-mentioned de-coupling process does not exist and results in complex mode shapes or, in simple terms, introduces phase differences which do not permit the mode to operate in simple harmonic motion. In this section, a physics-based illustration of the effect of non-proportionality is presented. *The reader may wish to skip the mathematics in this section and focus only on the text without losing continuity*.

If discrete viscous dampers are added, equation (1) is modified in a generic form as given below

$$M\ddot{u}(t) + C\dot{u}(t) + C_d\big(\dot{u}(t)\big)^{\alpha} + Ku(t) = f(t) \qquad (2)$$

Here, C is the inherent damping of the structure, assumed by the seismic codes to be proprtional, C_{d} is the added damping coefficient and $\boldsymbol{\alpha}$ is the velocity exponent normally ranging between 0.1 and 1.0.

For simplicity, let's also consider $\alpha = 1.0$. Then, equation (2) becomes,

$$M\ddot{u}(t) + [C + C_d]\dot{u}(t) + Ku(t) = f(t) \qquad (3)$$

For notational simplicity, let's assume $[C + C_{\mathrm{d}}]$ as simply C from here on.

For the structure presented in equation (3) to have *normal modes* or *classical modes*, the Caughey criterion needs to be satisfied. The Caughey criterion states that, for normal modes to exist (Caughey & Kelly 1965),

$$KM^{-1}C = CM^{-1}K \qquad (4)$$

In the case of equation (4), when discrete dampers are added ($C_{\mathrm{d}} \neq 0$), this does not hold true, in general, and hence the damping becomes non-proportional. The remaining part of this section will illustrate what this means and how it effects the dynamics of the damper-incorporated structure.

To understand physically the effects of non-proportionality, let's consider the homogenous form of equation (3):

$$I\ddot{u}(t) + M^{-1}C\dot{u}(t) + M^{-1}Ku(t) = 0 \qquad (5)$$

Equation (5) is further transformed as:

$$M^{-1/2}MM^{-1/2}M^{1/2}\ddot{u}(t) + M^{-1/2}CM^{-1/2}M^{1/2}\dot{u}(t) + M^{-1/2}KM^{-1/2}M^{1/2}u(t) = 0 \qquad (6)$$

In variable substituted form, Equation (6) becomes:

$$I\ddot{y}(t) + \hat{C}\dot{y}(t) + \widehat{K}y(t) = 0 \qquad (8)$$

Now, in the modal domain:

$$y(t) = \Phi_n q(t) \qquad (9)$$

Φ_n represents the undamped normal mode obtained by ignoring the damping term and $q(t)$ is the modal amplitude. Substituting equation (9) in equation (8) and pre-multiplying by ${\Phi_n}^T$, we get:

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}\{\ddot{q}(t)\} + [{\Phi_n}^T C \Phi_n]\{\dot{q}(t)\} + \begin{bmatrix} \widehat{k_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \widehat{k_n} \end{bmatrix}\{q(t)\} = 0 \tag{10}$$

If the damping is proportional, the second matrix term on the left-hand side should also have a diagonal form that de-couples the entire MDOF into a series of SDOFs. This is not the case when discrete dissipation devices are added into the structure as the Caughey criterion is generally violated and the damping non-proportionality phenomenon is exhibited. *In other words, systems with non-proportional damping will have off-diagonal terms which are of considerable magnitude preventing the above-mentioned de-coupling of the entire MDOF structure into a series of SDOFs.*

Therefore, an alternative treatment with damped eigen-decomposition needs to be undertaken. The eigen decomposition of the $M - C - K$ structure is performed through a state-space formulation.

$$\begin{Bmatrix} \ddot{u}(t) \\ \dot{u}(t) \end{Bmatrix} = \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix}\begin{Bmatrix} \dot{u}(t) \\ u(t) \end{Bmatrix} \tag{11}$$

$$\left.\begin{aligned} [A] &= \Phi\Lambda\Phi^{-1} \\ \Phi &= \begin{bmatrix} \Phi_c\Lambda_c & \Phi_c^*\Lambda_c^* \\ \Phi_c & \Phi_c^* \end{bmatrix} \\ \Lambda &= \begin{bmatrix} \Lambda_c & 0 \\ 0 & \Lambda_c^* \end{bmatrix} \\ \Lambda_c &= \begin{bmatrix} \lambda_{c,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{c,n} \end{bmatrix} \end{aligned}\right\} \tag{12}$$

Here, $\lambda_{c,i}$ are the eigen values and Φ_c is the complex mode shape matrix for $the\ M - C - K$ system.

The interesting question is now: What do we mean by a complex mode shape?

A simple explanation is that, when a structure exhibits a complex mode shape, different masses reach their maximum amplitudes at different times; in other words, there are phase shifts among different masses in each mode. Every component *t* of the mode shape is a complex number with real and imaginary components. This is illustrated in the numerical section in the mode shape plots where a comparison between a normal mode and a complex mode shape is given. This is very much in contrast to a normal mode where all masses reach their extreme displacements at the same instant - which allows the concept of effective mass and hence the de-coupling. In other words, the complex mode shape does not allow de-coupling of the MDOF structure into a series of SDOFs as there will be energy transfers happening between the modes. This phenomenon is termed as non-proportionality.

To further investigate the concept of non-proportionality, let's rewrite equation (11) in the complex modal domain, maybe written as,

$$\omega_i^2 + \omega_i \frac{(\phi_{n,i}^T \hat{C} \phi_{c,i})}{\phi_{n,i}^T \phi_{c,i}} + \omega_{n,i}^2 = 0 \tag{13}$$

Equation (13) may be re-written as:

$$\omega_i^2 + \omega_i \left(c_{ii} + \frac{(\phi_{n,i}^T \hat{C} \phi_{c,i})}{\phi_{n,i}^T \phi_{c,i}} \right) + \omega_{n,i}^2 = 0 \tag{14}$$

Where c_{ii} represents the real component of the damping coefficient and $\frac{(\phi_{n,i}^T \hat{C} \phi_{c,i})}{\phi_{n,i}^T \phi_{c,i}}$ is the imaginary component of the damping coefficient.

Since $\phi_{c,i}$ is a complex value, we can say that:

$$\frac{1}{2\omega_{n,i}}\left(c_{ii}+\frac{(\phi_{n,i}^{T}\hat{C}\phi_{c,i})}{\phi_{n,i}^{T}\phi_{c,i}}\right)=\xi+i\gamma \qquad (15)$$

In the spirit of the damping ratio in classical modal analysis, equation (15) describes the complex damping ratio. Alternatively, equation (15) may be viewed as a combination of the classical damping ratio (which is *real*) and an *imaginary* damping ratio component which reflects the *other mode effects* on the damping ratio. In other words, these so-called *other mode effects* create the phenomenon of *non-proportionality*.

To physically illustrate this, when non-proportional damping is present, it almost acts like the presence of an imaginary device which creates virtual modes. The net effect of this phenomenon is that a non-proportionally damped structure can never be de-coupled in the modal or physical domain. Trying to describe the dissipation effect using the classical damping ratio only (ignoring the *modal energy transfer*) may be a violation of physics and hence highly flawed. This is illustrated in examples in the numerical section for a simple four-storey structure.

3 VALIDITY OF PSEUDO STATIC METHODS FOR DESIGNING VISCOUSLY DAMPED SYSTEMS

Before delving into numerical aspects of complex modal analysis, this section conceptually reviews the validity of pseudo-methods for designing viscously damped systems. In seismic engineering, pseudo-static methods are very popular mainly because of their simplicity for application in a design office setup. Structural engineers are generally well-versed in static analysis and the thinking that goes with it. This is mainly because other types of loadings acting on the structure, such as wind, snow, gravity, etc., are predominantly treated as static forces. This comfort level in the understanding of static analysis methods and static line of thinking and that early seismic engineering was developed from wind engineering, necessitated development of static methods in seismic engineering approximating the dynamic effects in some sense. A considerable amount of research effort for more than six decades has been directed towards this aspect.

Mathematically, a structure subjected to an earthquake maybe represented by equation (1).

In simple terms, in most of the static methods, the mass and damping terms in (1) are set to zero with some adjustments to the effective load vector to mimic the effects of mass and damping. In other words, in a qualitative mathematical sense, most of the pseudo-static methods ignore the direct effect of mass and damping and try to reflect their affects by modifying the demand or imposed load vector $F_{\text{structure}}$ in an indirect manner. This modifies equation (1) as,

$$\left.\begin{array}{c}\{F_{\text{structure}}\}=-M\{\ddot{u}(t)+R\ddot{u}_{\text{g}}(t)\}+\{F_{\text{damping,5\%}}\}\\ or\\ \{F_{\text{structure}}\}=-M\{\ddot{u}_{\text{T}}(t)\}+\{F_{\text{damping,5\%}}\}\end{array}\right\} \qquad (16)$$

where $\ddot{u}_T(t)$ *is* the total ground acceleration.

There are simple and advanced ways of doing these modifications. The term on the right-hand side results in some of the most common pseudo-static methods presently used in design. If the term adopts a Newtonian form, it results in the Force-Based Method, a Hookean form results in Displacement Based Design and so on. Also, there are other forms which result in energy methods. The literature also has variations of these approaches, but the key assumptions for all these methods mostly remain the same. Only the common key assumptions inherent in the existing methodologies are listed in the following sub-section. No specific method is discussed in detail, though a conceptual critique of the assumptions is presented.

3.1 Key erroneous assumptions common to the existing pseudo-static methods

The key assumptions in whole or in part, presented in the majority of the pseudo static methods, are as follows:

1. *Assumption of an effective damping ratio based on assumed structure ductility*.

The main benefit is that it allows the whole structure dissipation to be characterised by a single number, structure ductility, which may be related to an equivalent damping ratio that maybe applied to the effective SDOF. The whole, complex MDOF dynamics are now simplified as simple SDOF statics. For this assumption to be valid, what is the expectation from the inelastic mechanics of the structure? The expectation is that hinges appear uniformly and are assumed to have equal dissipation power as shown in Figure 1.0.

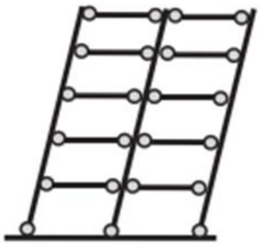

Figure 1: Idealised plastic mechanism assumed in pseudo static method

Now the pertinent question is whether the mechanism exhibited in Figure 1.0 is actually achievable in real life. As a case study, though not part of the present work, in Figure 2.0, we present the work of the 4th Author where snapshots of hinge migrations of a 13 storey structure are presented.

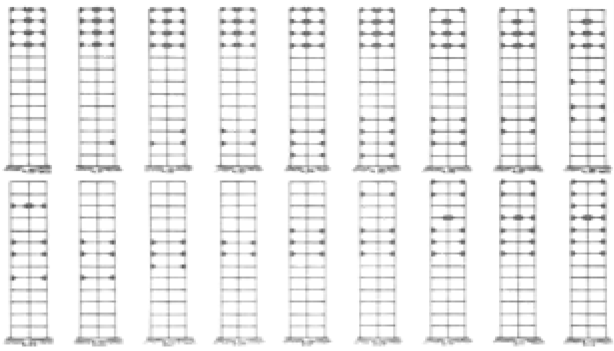

Figure 2: Hinge migration in a multi-storey structure when subjected to El-Centro May 1940

The black dots in the Figure 7 depict the plastic hinges and the numeral value below each frame depicts the time (i.e., 4.89 means 4.89 seconds). It can be clearly seen that in every snapshot the pattern of hinging is different. Similar studies have been done by Maniatakis et al. (2013). Also, each of those hinges in different snapshots will be in a different state of yielding - giving nonuniform dissipation capability. Figure 2 does not indicate the sign of the deformation in the plastic hinges. One observes in a dynamic analysis that a wave of plasticity propagates up the structure and reflects when it reaches the top. It can be observed that hinging can be in one direction in parts of the structure and in the opposite direction in other parts in the structure. This can be likened to higher-mode effects in the structural response. So, a choice of uniform/constant structure ductility for all modes (and hence, therefore, a choice of effective viscous damping ratio representing the hysteretic dissipation) exhibits a clear violation of physics.

2. *Conversion of a multi-degree-of-freedom structure (MDOF) to, effectively, a single-degree-of-freedom (SDOF) structure.*

Approximating a MDOF structure as a SDOF structure has been a primary assumption in seismic engineering and it is only possible, if damping proportionality with both mass and stiffness is assumed. A SDOF system is always proportionally damped though in real life there are no SDOF structures. Approximating a damped elastic MDOF structure with a SDOF structure is only possible if *proportionality* is assumed in the damping of the structure; in other words, the Caughey criterion described by equation (4) should be satisfied. So, by adopting a SDOF analytical representation, the Displacement-Based Design procedure is inherently assuming proportionality. In the penultimate section, in the numerical examples for damped structures, it is shown that the concept of proportionality is highly erroneous and hence the MDOF structure cannot be disaggregated into a set of SDOF structures. So, adopting methods relying on SDOF assumption for damper designs has no physical or mathematical basis other than its mathematical convenience.

3. *Assumption of a close-to-linear elastic mode shape to approximate an inelastic displacement shape.*

Most of the methods use a close-to-linear elastic mode shape in approximating the inelastic mode shape. This assumption is necessary in most of the methods to project the results obtained by the assumption of SDOF back into the MDOF domain. This assumption literally means that, when the structure goes inelastic, the "effective mode shape" does not change. Thus, in reviewing Figure 2, the pertinent question is whether or not this assumption is valid?

Firstly, let us review this in terms of an undamped structure. Figure 2 clearly illustrates that, at every instant, the plastic hinge distribution changes. In other words, this means that in every instant the effective mode shape should change. How much it changes is a function of the distribution of the hinges and what degree of elemental ductility they have incurred.

In the case of a damped structure this is more complex as the mode shapes are no longer *normal* and they have a complex component even if the system is elastic. Again, the complex mechanics for the damped structure is shown in numerical examples in a later section of the paper for two instants of secant stiffness. A graphical depiction of a complex mode is also presented in the same section which clearly illustrates the inherent error in assuming a standard real mode shape in projecting the SDOF response back to the MDOF response.

4. *De-coupling of the proportions of base shear attributed to the parent frame without the dampers and to the damper system itself.*

Now, writing equation of motion for a viscously-damped structure in Newtonian format:

$$M\{\ddot{u}(t) + R\ddot{u}_g(t)\} = -\left\{C\dot{u}(t) + C_{\mathrm{d}}\left(\dot{u}(t)\right)^{\alpha} + Ku(t)\right\} \tag{17}$$

In other words, in total acceleration format, we have,

$$M\{\ddot{u}(t) + R\ddot{u}_{\mathrm{g}}(t)\} = -\{F_{\mathrm{damping}} + F_{\mathrm{structure}}\} \tag{18}$$

Equation (18) illustrates that the net base shear is resisted by the damping component and the stiffness component. Now, let's compare this with a conventional structure. For a conventional structure, F_{damping} is assumed to be small (in a SDOF structure,5% critical viscous damping produces damping forces of the order of 10% of the other forces in the structure) and, in a classical seismic analysis framework, Equation (18) gets modified to:

$$\left.\begin{array}{c}\{F_{\mathrm{structure}}\} = -M\{\ddot{u}(t) + R\ddot{u}_{\mathrm{g}}(t)\} + \{F_{\mathrm{damping,5\%}}\} \\ or \\ \{F_{\mathrm{structure}}\} = -M\{\ddot{u}_{\mathrm{T}}(t)\} + \{F_{\mathrm{damping,5\%}}\}\end{array}\right\} \tag{19}$$

where $\ddot{u}_{\mathrm{T}}(t)$ *is* the total ground acceleration. So, in a conventional system, the net inertial seismic force is modified by the allowance from damping (which is usually the code based 5% modal damping) and is resisted by the structure.

In a structure with added dampers, the whole of Equation (18) becomes more complex as F_{damping} is a larger quantity exhibiting complex modal mechanics and the whole structure is highly coupled.

It should be noted that in real life there are no single degree-of-freedom structures.

To ease the design process for the application of the viscous device and to adapt the complex dynamic design into the well-established *static displacement-based framework*, some of the building codes adopt an arbitrary split of the base shear between the F_{damping} and $F_{\mathrm{structure}}$. One such classical approach adopted by the ASCE codes is the 70%-30% split of the base shear for symmetric structures, where 70% is taken by the structure and 30% is taken by the dampers. *This split has no scientific merit or physical evidence other than the mathematical convenience, but it provides a means to adopt a displacement-based approach for the design of a structure with velocity-based devices*. Refer to the penultimate section for numerical evidence. An ad hoc reason that may be given for the adoption of such a split may be the realisation that there is a need for a certain amount of structural stiffness required for the dampers to work against.

4 NUMERICAL METHODS

This section investigates the validity of the pseudo-static methods for inelastic structures with dampers using the rigorous nonclassical modal dynamics. It also illustrates the mechanics of *damper-structure interaction*.

In this process we look to answer the following question:

Are modal methods/modal-based, pseudo-static methods applicable for either a linear elastic structure or an inelastic structure if it is fitted with viscous dampers?

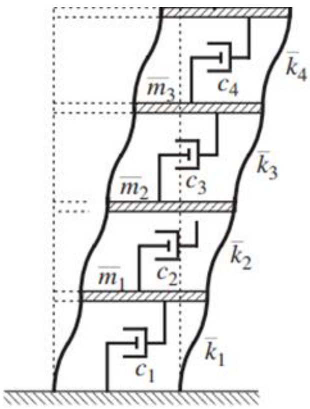

Figure 3: Four storey shear frame

The structure in Figure 3 is used. The mass and the stiffness properties remain the same, and damping is attributed to discrete dashpots. Two scenarios exist: one with damping distributed to match the stiffness or the mass and one with non-uniform damping. The first scenario matches the Rayleigh damping model and will then have a very similar behaviour to the one exhibited by classical modal dynamics. This approach is not commonly used and is not of any interest in this paper. Here, only the mechanics associated with non-uniform damping are discussed. For practical, real structures, it is more usual to arrive at a non-uniform damping in a Performance-Based Design framework. Three snapshots in time of the stiffness matrix, very similar to those adopted earlier (refer Figure 2), are investigated.

For the case of non-uniform damping, only the damper coefficients at Levels 1 and 2 are assumed to have a non-zero coefficient value and the rest are assumed to be zero; $c_1 = 2 \times 10^4\ N - s/m$ and $c_2 = 1 \times 10^5\ N - s/m$.

The Caughey and Kelly criterion is violated and the structure is inherently non-proportionally damped.

Damping ratio

Table 1 records the damping ratio for the three different states of the system. The most unique feature for nonuniform damping is that, even when the structure remains elastic, the damping ratio per mode is a complex number; in other words, the structure is non-proportional - even in the elastic state.

It can also be seen that, depending on the degree of inelasticity, the non-proportionality increases. A feature of Table 5 is that it explicitly shows how complex the overall *damper-structure interaction* phenomenon is when dampers are added to the structure. Therefore, using simplified techniques to design such a complex structure needs to be reviewed rigorously.

Table 1: Damping ratio comparison for nonuniform damping for three different states of the system

Mode	Elastic	Inelastic 1	Inelastic 4
1	0.13 - 1.156*i*	0.20+ 1.682*i*	0.09 - 0.76*i*
2	0.07 - 1.212*i*	0.20- 1.866*i*	0.035 - 0.78*i*
3	0.63 + 0.019*i*	0.66 + 0.026*i*	0.06 + 0.09*i*
4	0.02 + 0.058*i*	0.02 + 0.055*i*	0.62+ 0.09*i*

Frequencies

Table 2 compiles the frequencies of the damped structure in all the 3 states. The most important point to be noted is that as the damping is non-proportional even in the elastic state, the frequencies are also complex numbers. So, *how can one apply spectral approaches to such a structure even in the elastic state?*

Table 2: Modal frequencies for three different states of the structure with nonuniform viscous dampers.

Mode	Elastic	Inelastic 1	Inelastic 4
1	1.97 + 2.151*i*	1.53 + 1.69*i*	1.94 + 2.09*i*
2	2.58 + 5.36*i*	2.34 + 5.168*i*	2.60 + 5.34*i*
3	3.34 + 3.50*i*	3.07 + 3.39*i*	2.83 + 2.90*i*
4	4.44 + 4.52*i*	4.43 + 4.52*i*	3.80 + 3.98*i*

Mode shapes

Figures 4 to 6 depict the mode shapes of both the elastic structure and the inelastic structure with nonuniform damping. As it was shown above in Tables 1 and 2, since even the elastic structure exhibits nonproportional characteristics, and to benchmark the nature of change in mode shape, a plot of the elastic structure with proportional damping (red) is also given.

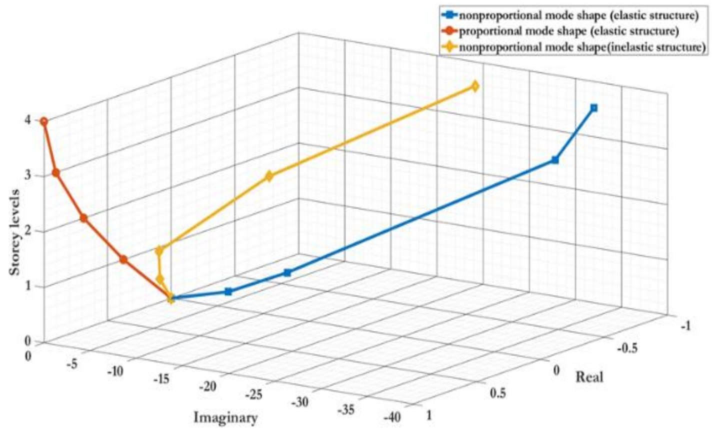

Figure 4: First mode (proportional mode vs. nonproportional)

From the plots it can be clearly seen that, even in the elastic case, the nature or shape of the mode is different - exhibiting phasing effects as different lumped masses reach their maximums at different instants. From this it is very clear that using a constant close to a linear-mode-shape assumption in some of the simplified methods in codes is quite misguided and violates the physics of the system. The proportional mode shape only has ordinates in the real axis and its imaginary component is completely zero whereas the non-proportional mode shape in both cases (elastic and inelastic state of the system) has ordinates in the imaginary axis. A design methodology assuming a mode shape that ignores the imaginary component will incur errors depending on the nonproportionality and inelasticity. Also, the modal characteristics such as the participation, etc., which drives the base shear will also be in highly erroneous.

Deriving an effective SDOF from the MDOF using such a physically inconsistent mode shape will clearly violate the physics of the reduced-order model. Again, designs obtained through analysis or design methodologies based on such pseudo approaches need to be thoroughly investigated. More on this is given in the next section.

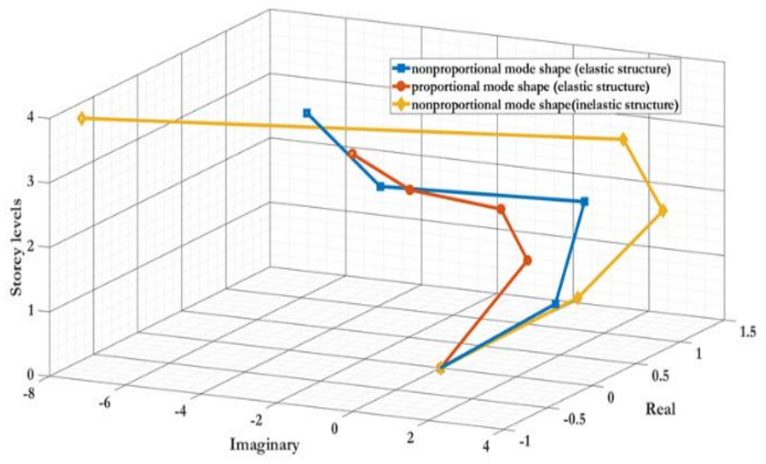

Figure 5: Second mode (proportional mode vs. nonproportional)

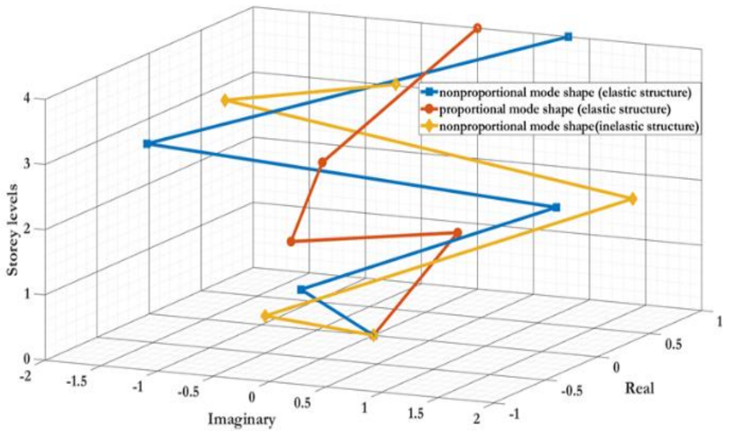

Figure 5: Third mode (proportional mode vs. nonproportional).

5 SUMMARY STATEMENT AND A POSSIBLE PATHWAY

As evidenced in the above sections, this paper raises some important questions about some of the fundamental assumptions inherent in seismic design for the past 60 years. To summarize, based on the studies presented in the previous sections of this paper, the following questions are raised concerning the pseudo-static and modal approaches currently in common use by structural designers:

a) *Can modal/pseudo static spectra-based methods be really applied to an inelastic or non-proportionally damped structure where they have complex natural frequencies? The code response spectra assume real frequencies.*

b) *Is there any mathematical or physical basis for the SDOF reduction of an inelastic MDOF or non-proportionally damped MDOF structure based on the use of elastic mode shapes?*

c) *Can modal methods, as advocated in structural building codes, capture the phenomenon of damper-structure interaction?*

d) *Is there any validity in the concept of an effective added-damping ratio both in inelastic and non-proportionally damped structures as implied by the used of the Displacement-Based design methods?*

e) *Is there any reliable definition of structural ductility in the case of 3D MDOF structures?*

f) *Is there any justification in the continual use of the equal displacement theory?*

g) *Are we justified in still blindly accepting the use of a 5% critical damping ratio that is assumed by most seismic codes in the production of the design spectra, particularly when we are modelling the inelastic energy dissipation effects directly?*

These questions demand a paradigm shift in design thinking. The authors do understand that raising all these issues is not a solution for going forward. The interesting aspect that the authors want to point out is that none of these questions arise if the design is based on nonlinear time-history analysis (NLTHA) by direct integration. The entire complex mechanics described earlier are inherently incorporated when NLTHA with direct integration is used as the analysis tool. However, there are practical difficulties and, even today, the authors do acknowledge that to design by time-history analysis in a conventional practice is often impractical both in terms of time and cost.

Proportional damping, which is a mathematical construction to allow the mathematical representation of a MDOF structural model as a SDOF structural model, is no longer valid when the structure goes inelastic or dampers are introduced and the resulting error introduced can be significant. Proportional damping is also extremely unlikely to be valid when the damping is represented at the element level rather than at the structural level. Direct integration analysis (NLTHA) is the only way to track the dependencies of the inelastic actions in structural members and the behaviour of the dampers. The use of time-history analysis becomes even more critical if the dampers are non -linear with respect ($\alpha \neq 1$) to their input velocity.

When we know that so many of the methods and assumptions used in current design are, at best, of debatable validity, are we still prepared, as a profession, to carry on as we have for the past several decades. Just because a method is popular, or commonly used, does not mean that is correct.

One of the major concerns with much of the simplified current design methods is how far are we from meeting the Swiss Cheese criterion where all the holes align.

An alternative approach would be to do the conceptual design by appropriate pseudo-static approaches and then verify the design using NLTHA methods. This can be done for both conventional structures and for those with added dampers. The authors do recognize that these are not ground-breaking ideas and are known to many design practitioners and researchers. However, the evidence presented in the previous sections literally begs the designer to adopt this in *a mandatory fashion to ensure resilience,* and also to make sure that the design intent is met, especially when the design is done for highly-seismic regions such as Wellington. In this way it can be ensured that all the aims made in the conceptual design are achieved and that the design achieves the intended performance. This thinking has been recognized before by practitioners and academics nationally and internationally (Engelkirk 2003).

The authors are fully aware that the NLTHA approach is not without its difficulties. No method of analysis can be any more accurate than the lowest accuracy of any data that is used as input. There are the problems of uncertainties in material properties, member section data, plasticity modelling, hysteresis modelling, applied loads and the selection of the earthquake excitations and their scaling. The major drawback to using such analyses will be in modelling the structure, in data preparation and in the evaluation of the analysis results. However, such NLTHA are the only way to evaluate the inelastic behaviour of the members of the structure and/or the effects of dampers added to the structure. This non-linear behaviour is not able to be replicated by any of the approximate methods of analysis. Even the supposed accuracy of the traditional modal analysis, as proposed by many of the seismic codes cannot represent these effects because the very superposition implied in combining the modes precludes any non-linearity in member behavior.

This alternative approach is very important for structures incorporating dissipation devices. As these devices inherently affect the dynamics of the structure, the forces of the devices, the connection forces and attributed stresses in all elements should be estimated from inelastic dynamic analysis using time-marching schemes rather than from pseudo-static modal approaches or other ad-hoc approaches presently adopted in the profession.

6 CONCLUSIONS

We feel that the NLTHA methods should be made mandatory for seismic design in regions with likelihood of large seismic shaking. Though the main focus of the study has been on damped structures the above statement is valid for conventional systems as well. The results from the simplified techniques that may be satisfactory for preliminary design need to be verified by more rigorous analyses techniques.

References

Chopra AK. (2017) "Dynamics of Structures" *5th Ed..Pearson, Hoboken, NJ. 960p*

Clough RW. and Penzien J. (1993) "Dynamics of Structures", *2nd Ed. McGraw-Hill, NY. 738p*

Engelkirk, RE. "Seismic Design of Reinforced and Precast Concrete Buildings". *Wiley 2003*, p848

Maniatakis CA., Psycharis IN, Spyrakos CC. (2013) " Effect of higher modes on the seismic response and design of moment-resisting RC frame structures" *Engineering Structures*:56, 417-430

Veletsos AS and Ventura CE (1986). " Modal analysis of non-classically damped linear systems" *Earthquake Engineering Structural Dynamics, Vol 14, Issue 2*.

Hurty WC. and Rubinstein MF. "Dynamics of Structures", *Prentice-Hall, Englewood Cliffs N.J. 1964, 455p*