Area Ratio or Volumetric Ratio: which is better for quantifying the confinement of concrete?

D.K. Jaiswal & C.V.R. Murty

Department of Civil Engineering, Indian Institute of Technology Madras, Chennai 600036, INDIA

ABSTRACT

Confinement of concrete helps in improving ultimate strain of concrete. This increase depends on amount of transverse reinforcement in the vertical member. While 135° hooks are mandatory in transverse reinforcement, the distribution of transverse steel along the two principal plan directions of the vertical member influences the increase in the ultimate strain of concrete. The *volume* of transverse reinforcement does not reflect the actual distribution of transverse reinforcement along the two plan directions. The amount of transverse reinforcement provided along the two plan directions can be considered in terms of either *volumetric ratio* or *area ratio*. Literature suggests using these two options for quantifying the effects of confinement of concrete. The matter is more critical in RC columns with *hollow* cross-sections than in those with *solid* cross-sections. This paper presents an analysis in favour of the use of *area ratio* when quantifying the amount of transverse reinforcement to be provided in RC columns.

1 INTRODUCTION

The stress-strain curve of plain concrete under uniaxial compression depends on its grade. This σ–ε curve can be defined fully with four critical parameters, namely its compressive strength f_{c0}, strain ε_{c0} corresponding to peak stress f_{c0}, ultimate strain ε_{cu}, and geometries of ascending and descending curves (Fig. 1). These parameters are enhanced when concrete is placed in reinforced concrete (RC) members along with closed-loop transverse reinforcements with 135° hooks (Fig. 1). When a RC member is compressed vertically, concrete dilates outwards in the plan transverse directions owing to the *Poison's Effect*. This outward dilation is restrained by the transverse reinforcement owing to bond between concrete and these transverse steel closed loops; as a consequence, additional compressive stresses are induced in concrete along the transverse plan directions (Fig. 1). These additional transverse stresses (*confining stresses*) in the *confined concrete* provide confining action and enhance the said critical properties of concrete, namely compressive strength f_{cc}, strain ε_{cc} corresponding to peak stress f_{cc}, ultimate strain ε_{ccu}, and geometries of ascending and descending curves of *confined concrete* (Fig. 1).

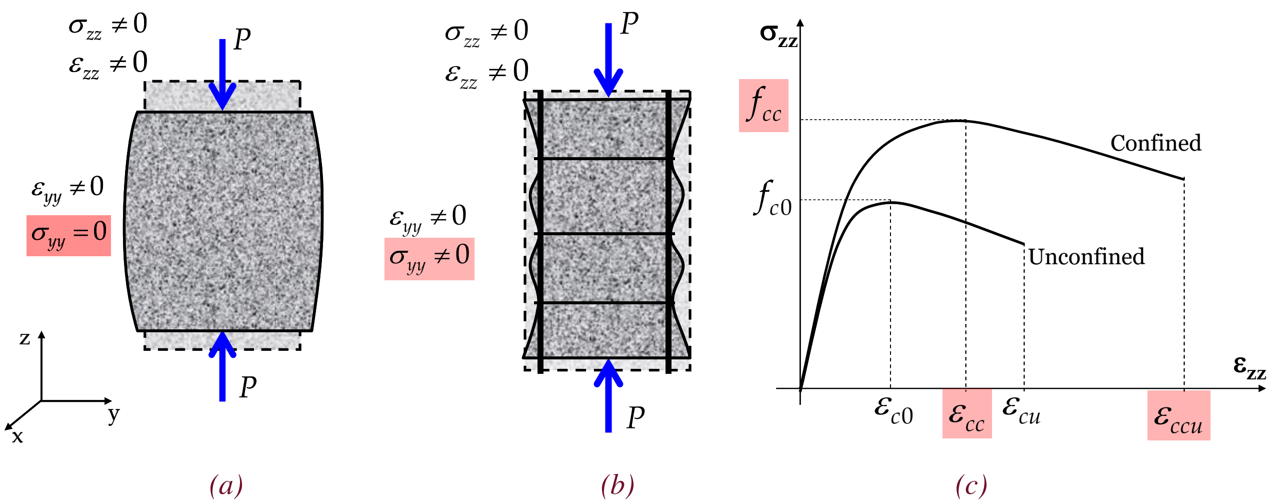

Figure 1: Dilation of concrete under uniaxial compression; (a) without confining reinforcements, & (b) with confining reinforcements; and (c) Stress-strain curve of confined and unconfined concretes

The extent of the confining of concrete inside the transverse reinforcement depends on the four properties of transverse reinforcement having mandatory 135° hooks, namely diameter of bars, vertical spacing and in-plan distribution within the cross-section. To quantify the extent of confinement provided by transverse reinforcement, the methods presented in literature reflect it as a function of *volumetric ratio* or of *area ratio*; volumetric ratio is used more often. This paper presents a discussion on relative merits of these two ratios.

2 PHILOSOPHY OF AREA RATIO AND VOLUMETRIC RATIO

The lateral dilation of concrete is resisted by the transverse reinforcements. Strain compatibility exists between concrete and steel. Hence, total tensile force induced in transverse reinforcement (because of transverse dilation of concrete) is equal to the total compressive confining force induced in confined concrete (Fig. 2), and:

$$\sum_{i=1}^{N_{sv}} f_{si} A_{svi} = f_{le} A_c \,, \tag{1}$$

where f_{si} = tensile stress induced in the transverse steel, f_{le} = confining compressive stress induced in core concrete, A_{svi} = cross sectional area of transverse steel bar i, A_c = plan cross-sectional area of member. This implies that higher the amount of confining reinforcement provided along the direction of dilation higher will be the confinement in the concrete. This amount of reinforcement can be quantified either by *volumetric ratio* or by *area ratio*.

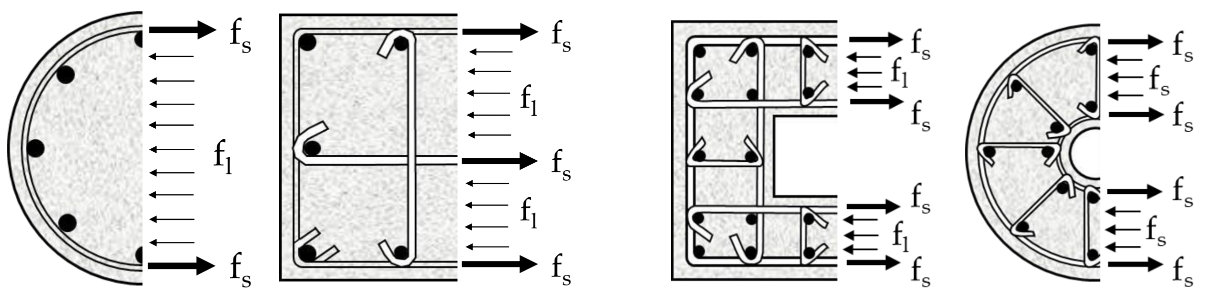

Figure 2: Equilibrium of forces in one confining direction

2.1 Volumetric Ratio

It is the ratio of volume of transverse reinforcement at any level to that of concrete between the centers of two adjoining transverse reinforcement sets in the vertical direction (Fig. 3a). This ratio accounts for the volume of all steel placed in any direction in plan. But, bars placed in any direction may not be fully effective to resist the dilation of concrete, as is evident from force equilibrium in transverse direction (Eq.1).

2.2 Area Ratio

It is the ratio of total cross sectional area of transverse reinforcement at any level to that of section of concrete between the centers of two adjoining transverse reinforcement sets perpendicular to the direction of dilation (Fig. 3). Figure 3 shows geometries used in estimating *volumetric* and *area ratios* in a rectangular RC column with 3 legged transverse reinforcements in along each plan direction.

2.3 Literature Survey

Volumetric ratio is used commonly in literature to quantitatively estimate the confinement of concrete. The initial studies conducted on confinement of concrete use *volumetric ratio* to quantify the effects of transverse reinforcements [Sheikh and Toclucu, 1993; Azizinamini and Kuska, 1994; Mandal 1993; Samra et al., 1996; Xie et al., 1997; Chan 1955; Blume et al., 1961; Kent and Park, 1971; Sheikh and Uzumeri, 1982; Sheikh and Yeh, 1992; Park and Leslie, 1977; Desayi et al., 1978; Ahmad and Shah, 1982, 1985; Young et al., 1988; Dilgar et al., 1984; Hoshikuma et al., 1997; Cusson and Paultre, 1995; Han et al., 2003; Hong et al., 2006; Bing 2001; Suzuki et al., 2004; Chung 2002]. *Area ratio* is used exclusively only in few studies [Saatcioglu and Razvi, 1992; Razvi and Saatcioglu, 1999a; Mander et al., 1988a; Nagashima et al., 1992; Scott et al., 1982; Sharma et al., 2005; Dasgupta 2000]. But, even those studies use *volumetric ratio* to estimate the maximum strain of the confined concrete. Even the recent studies [Hou 2020; Wu et al., 2018; Chang et. al., 2021; Eid et al., 2018; Weilun et al., 2017; Awati and Khadiranaikar, 2012; Lee et al., 2016; Shin et al., 2014;] continue to use *volumetric ratio* as the basis for quantifying transverse reinforcements.

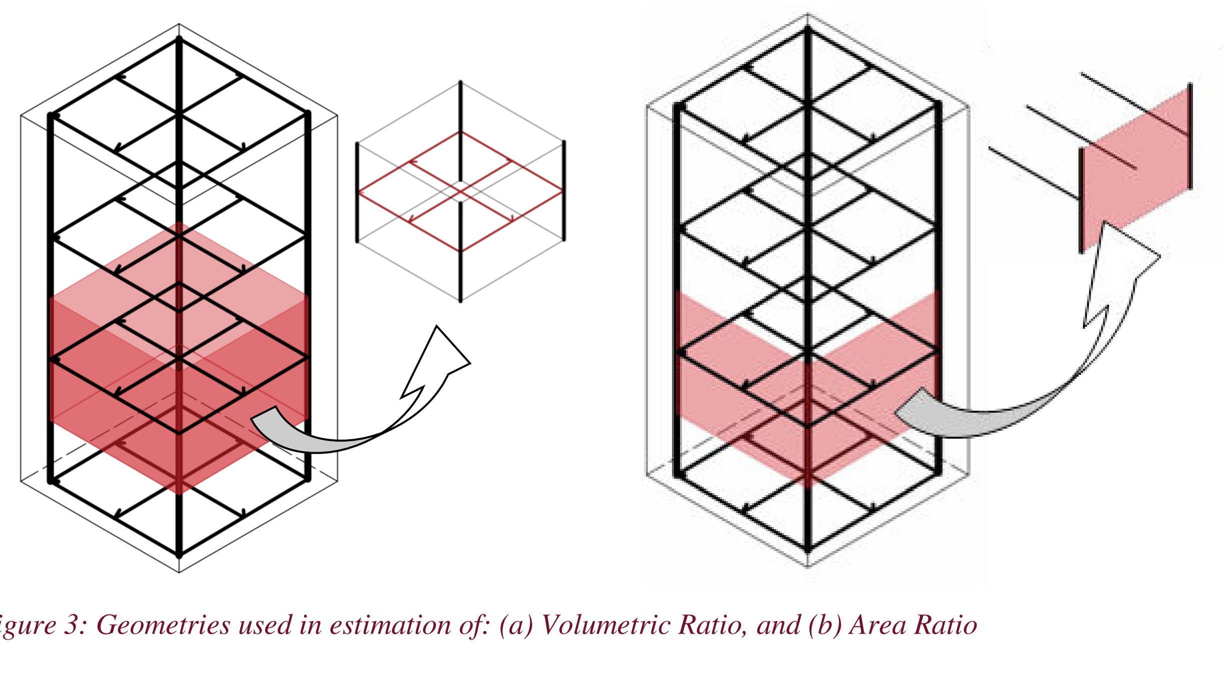

Figure 3: Geometries used in estimation of: (a) Volumetric Ratio, and (b) Area Ratio

3 ANALYTICAL STUDY

The *area ratio* and *volumetric ratio* of rectangular and circular sections (both solid and hollow cross-section) are shown in the Eqs.2-9. Here, closed-loop transverse reinforcements (in cross-ties and links) are considered with the expected detailing (Fig. 4).

3.1 Rectangular Sections

3.1.1 Solid Section

Volumetric Ratio $\rho_v = \dfrac{N_{lx}A_{svx}D_c + N_{ly}A_{svy}B_c}{B_c D_c s_v}$, and (2)

Area Ratio along x-direction $\rho_{ax} = \dfrac{N_{lx}A_{svx}}{B_c s_v}$ and Area Ratio along y-direction $\rho_{ay} = \dfrac{N_{ly}A_{svy}}{D_c s_v}$, (3)

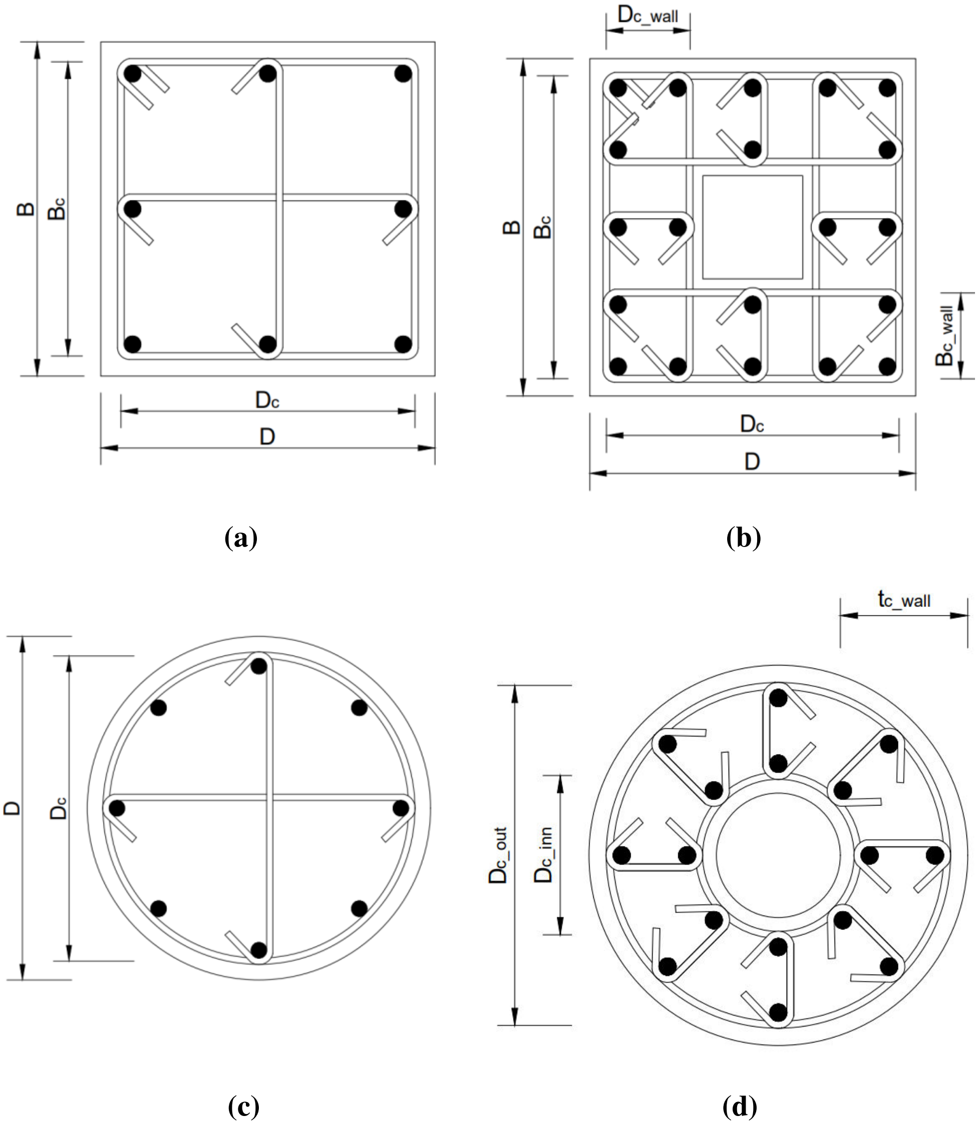

Figure 4: Cross-sections considered and notations adopted in: (a) Solid Rectangular Sections, (b) Hollow Rectangular Sections, (c) Solid Circular Sections, and (d) Hollow Circular Sections

where s_v is the vertical spacing of transverse steels; N_{lx} and N_{ly} the number of legs cutting the cross-sections perpendicular to the transverse steel along x- and y-directions, respectively, and A_{svx} and A_{svy} the area of each transverse steel along x- and y-directions, respectively. In a rectangular section with transverse reinforcement placed along the two plan orthogonal directions, *volumetric ratio* is the sum of the *area ratios* along the two plan directions. But, this is not true with the diamond-shaped stirrups.

3.1.2 Hollow Section

$$\rho_{ax} = \frac{4A_{svx}}{2B_{c\_wall}S_v}; \rho_{ay} = \frac{4A_{svy}}{2D_{c\_wall}S_v} \tag{4}$$

$$\rho_v = \frac{4A_{sv}D_c + 4A_{sv}B_c + 2N_{lwx}A_{sv}D_{c\_wall} + 2N_{lwy}A_{sv}B_{c\_wall}}{\{2D_cB_{c\_wall} + 2(B_c - 2B_{c\_wall})D_{c\_wall}\}S_v} \tag{5}$$

where N_{lwx} and N_{lwy} is number of legs of cross ties along the x- and y-directions in plan, respectively.

In hollow sections, *volumetric ratio* accounts for all transverse reinforcements *together*, but *area ratio* accounts for transverse reinforcements independently (web alone, flanges alone, or both). The least confined zone in whole section governs the overall capacity of the section, because concrete will dilate from the least confined zone. In *area ratio*, this clarity and distinction is possible (term 4 in the Eq. 4).

3.2 Circular Sections

3.2.1 Solid Section

$$\rho_v = \frac{A_{sv}\pi D_c + A_{sv}D_c(N_l - 2)}{\frac{\pi}{4}D_c^2 s_v} \quad \text{and} \tag{6}$$

$$\rho_a = \frac{N_l A_{sv}}{D_c s_v} \tag{7}$$

where N_l = number of cross links, and A_{sv} = area of each cross link. As in rectangular sections, the *volumetric ratio* is two times that of *area ratio*, if the cross-links are absent or not considered.

3.2.2 Hollow Section

$$\rho_a = \frac{4A_{sv}}{2t_{c\_wall}S_v} \tag{1}$$

$$\rho_v = \frac{N_l A_{sv} t_{c\_wall} + A_{sv}\pi(D_{c\_inn} + D_{c\_out})}{\frac{\pi}{4}\left(D_{c\_out}^2 - D_{c\_inn}^2\right)S_v} \tag{2}$$

In hollow circular sections, radial links provide confinement only when the $D_{inner}/D_{outer} <\cong 0.6-0.7$; columns with $D_{inner}/D_{outer} > 0.6-0.7$ are not recommended, because both inner and outer perimeters dilate outwards making the concrete unconfined. For $D_{inner}/D_{outer} <\cong 0.6-0.7$, the confinement is dependant only on the circumferential hoops; links improve wall behaviour, but do not participate in overall confinement of concrete. This discussion is relevant only for area ratio; the discussion does not arise in volumetric ratio, because volumetric ratio does not reflect the confined behaviour of the column and its wall thickness.

4 PROPOSAL

The effect of confinement of concrete by transverse reinforcement in plan cross-section together is modelled using translational springs (Fig. 5). The stiffness of the spring in each direction is that of the sum of the stiffness of individual transverse steel bars. The stiffnesses of these springs are combined (considering it to be a series connection) to obtain an equivalent spring stiffness [Jaiswal 2023]. Here, the stiffnesses of the transverse steels in the two plan directions are proportional to the corresponding *area ratios* (Fig. 5). An *effective area ratio* ρ_{ae} of confinement of concrete is estimated by combining the effects of confinement along each plan direction, given by:

$$\rho_{ae} = \frac{\rho_{ax}\rho_{ay}}{\rho_{ax} + \rho_{ay}}, \tag{10}$$

where ρ_{ax} and ρ_{ay} are the area ratios along the plan x- and y-directions, respectively. Hence, combining the area ratios in the two orthogonal directions gives an effective confinement ratio of the entire section.

Such a combination is not possible when *volumetric ratio* is considered to quantify confinement of concrete by transverse steel reinforcements. It was shown in section 3 that *volumetric ratio* is equal to the sum of *area ratios* in the two orthogonal plan directions in rectangular sections; but, this direct sum does not consider different confinement along the two plan directions. For example, in structural walls, hollow sections and columns with different shear demand along the two plan directions, the transverse reinforcement in the two orthogonal plan directions can be different.

The variation in ρ_{ae} with ρ_{ax} and ρ_{ay} (Fig. 6) provides two important physically intuitive lessons:

(1) The curve is nonlinear and saturating with increase in the area ratio along any one plan direction; and

(2) The effective confining area ratio is governed by the least of the two area ratios. Also, if the area ratio in any one direction is zero, the effective area ratio becomes zero. Physically, if transverse reinforcement is missing in any plan direction, concrete will dilate more towards least confined direction, and hence have no confinement.

A comparison is drawn of the definitions of effective confinement offered by transverse steel as per the methods available in literature – the volumetric ratio [Hou 2020, and many others], and the proposed effective area ratio. Volumetric ratio varies linearly with transverse reinforcement in one direction, while the proposed effective area ratio varies asymptotically (Fig. 7). Similar observation is visible in strength enhancement by the popular expressions given in literature [Mander 1988]. The other popular method [Razvi and Saatcioglu, 1999], which uses weighted average of the confining stresses, follows a linear relation (Fig. 8). These two methods do not provide a physical feel of the influence of confinement offered by the transverse steel in the two plan directions. Even though these definitions vary, the final empirical expressions used in estimating the enhanced confined strengths of concrete are tuned (with empirical coefficients) to match the corresponding experimental values (Fig. 8).

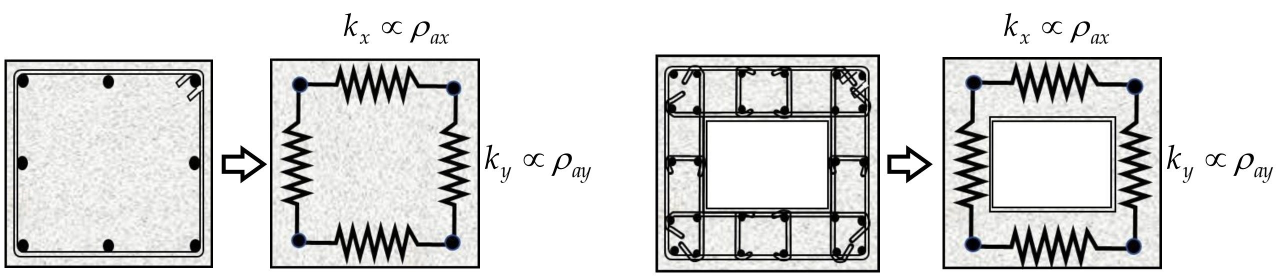

Figure 5: Modelling of transverse reinforcements as a translational spring

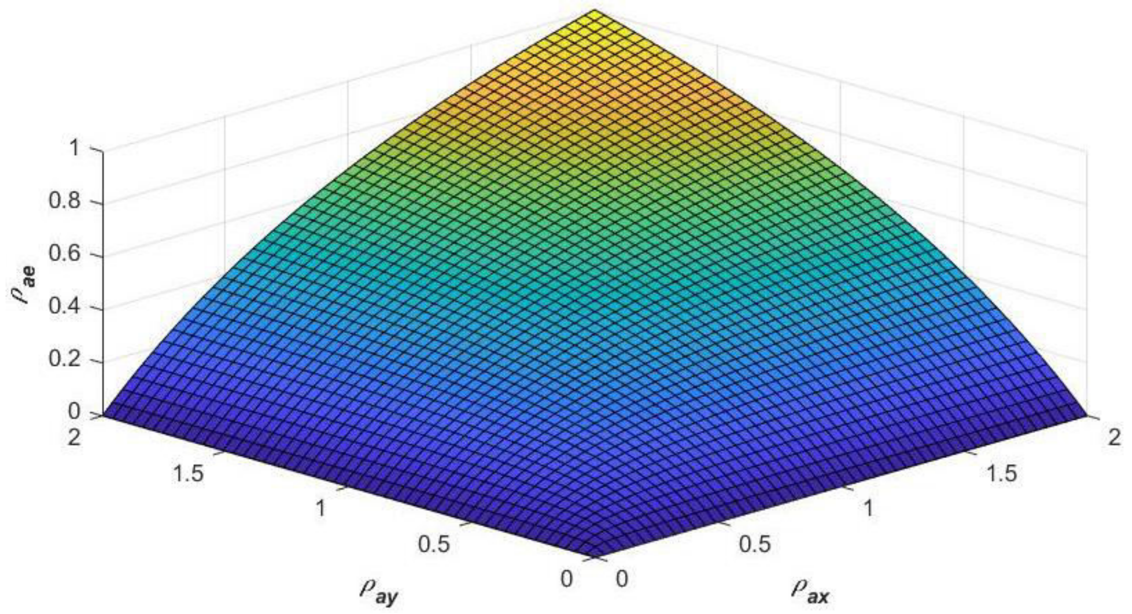

Figure 6: Variation of the effective area ratios

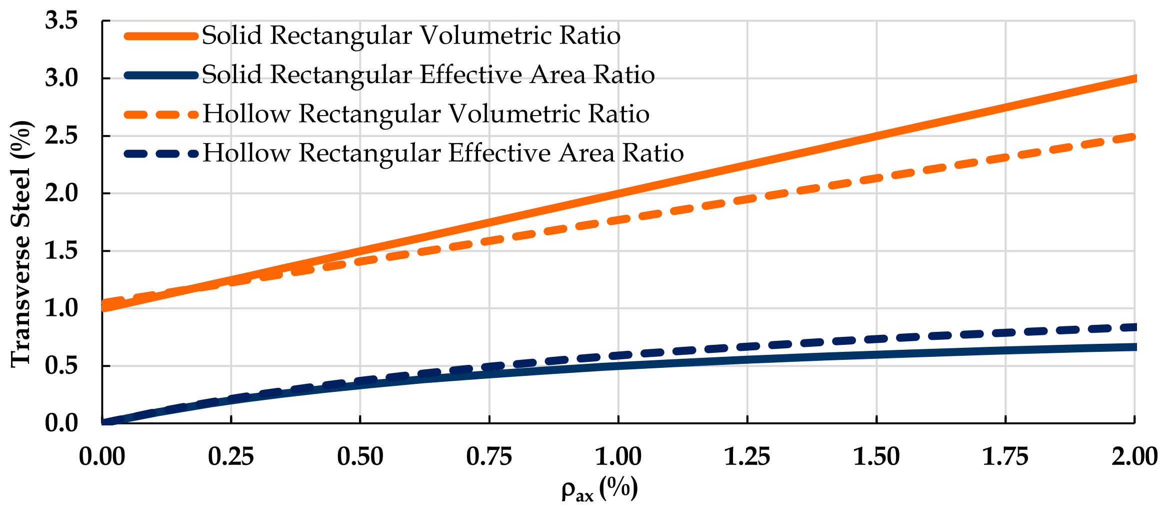

Figure 7: Influence of amount of transverse steel along one plan direction on confinement of concrete reflected by volumetric ratio and effective area ratio, in solid (ρ_{ay}=1%, varying ρ_{ax}) and hollow rectangular (ρ_{ay}=1.44%, varying ρ_{ax}) sections

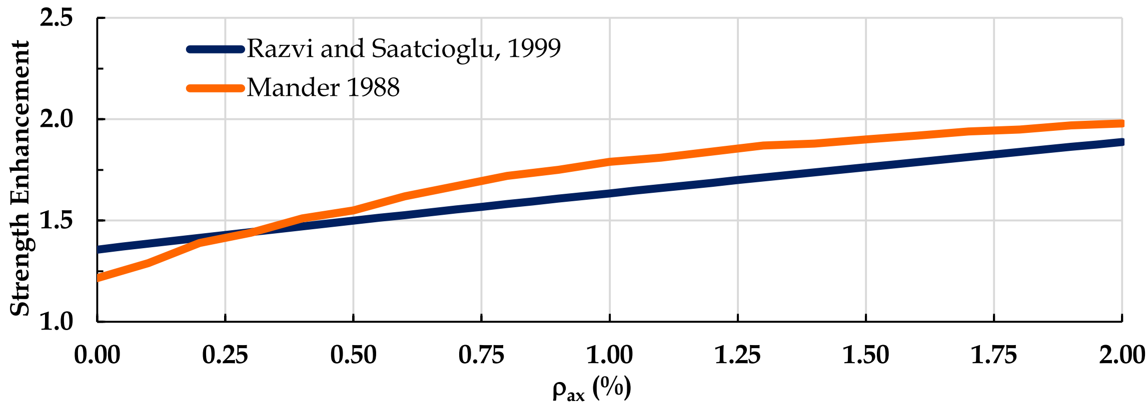

Figure 8: Influence of amount of transverse steel along one plan direction on confinement of concrete reflected by strength enhancement (ρ_{ay}=1%, varying ρ_{ax})

5 CONCLUSIONS

This paper discusses *volumetric ratio* and *area ratio* as parameters to quantify the effect of confinement offered by transverse steel reinforcement in vertical RC members. The salient conclusions of the study are:

(1) *Area ratio* identifies the transverse steel bars that are effective in confinement of concrete; this is not possible with *volumetric ratio*. In most cases, the volumetric ratio comes out to be the direct sum of the area ratio in the two orthogonal directions but the direct sum of the two may not be proper way of combination.

(2) Area ratio gives a better sense of understanding of confining action, because it considers *equilibrium of forces*, *constitutive law* and *compatibility*.

(3) Combining the area ratios in two orthogonal directions by assuming them as a series of springs is able to capture the effective confining reinforcement.

The above conclusions will help to: (a) improve estimation of confinement of concrete by transverse steel, (b) obtain the complete stress-strain curve of confined concrete, and (c) capture confinement of hollow circular and rectangular sections considering the effects of wall thickness and cross links.

REFERENCES

Azizinamini A, and Kuska S (1994). "Seismic Behaviour of High Strength Concrete Columns," *Proceedings of the 5th US National Conference on Earthquake Engineering*, Chicago, USA.

Awati M and Khadiranaikar RB (2012). "Behavior of Concentrically Loaded High Performance Concrete Tied Columns". *Engineering structures*, **37**: 76-87. https://doi.org/10.1016/j.engstruct.2011.12.040

Ahmad SH, and Shah SP (1982). "Stress-Strain Curves of Concrete Confined by Spiral Reinforcement". *ACI Structural Journal*, **79**(6): 484-490. https://doi.org/10.14359/10922

Ahmad SH, and Shah SP (1985). "Behaviour of Hoop Confined Concrete under High Strain Rates". *ACI Structural Journal*, **82**(5): 634-647. https://doi.org/10.14359/10373

Bing L, Park R and Tanaka H (2001). "Stress-Strain Behaviour of High-Strength Concrete Confined by Ultra-High and Normal-Strength Transverse Reinforcements". *ACI Structural Journal*, **98**(3):395-406. https://doi.org/10.14359/10228

Blume JA, Newmark NM, and Corning LH (1961). "Design of Multistorey Reinforced Concrete Buildings for Earthquake Motions". *Portland Cement Association*, Chicago, USA, 4, 318pp.

Chan WW (1955). "The Ultimate Strength and Deformation of Plastic Hinges in Reinforced Concrete Frameworks". *Magazine of Concrete Research*, **7**(21):121-132. https://doi.org/10.1680/macr.1955.7.21.121

Chung HS, Yang KH, Lee YH and Eun HC (2002). "Stress–Strain Curve of Laterally Confined Concrete". *Engineering Structures*, **24**(9): 1153-1163. https://doi.org/10.1016/S0141-0296(02)00049-4

Cusson D, and Paultre P (1995). "Stress-Strain Model for Confined High Strength Concrete". *Journal of Structural Engineering*, **121**(3): 468-477. https://doi.org/10.1061/(ASCE)0733-9445(1995)121:3(468)

Chang W, Hao M and Zheng W (2021). "Strength and Ductility of Laterally Confined Concrete". *Structural Concrete,* **22**(5): 2967-2991. https://doi.org/10.1061/(ASCE)0733-9445(1992)118:6(1590)

Dasgupta P, (2000). "Effect of confinement on strength and ductility of large RC hollow sections". Masters Thesis, Department of Civil Engineering, Indian Institute of Technology Kanpur, India.

Desayi P, Iyengar KTSA, and Reddy TS (1978). "Equations for Stress-Strain Curve of Concrete Confined in Circular Steel Spiral". *Materials and Structures,* **11**(65): 339-345. https://doi.org/10.1007/BF02473875

Dilger WH, Koch R, and Kowalczyk (1984). "Ductility of Plain and Confined Concrete under Different Strain Rates". *ACI Structural Journal*, **81**(1): 73-81. https://doi.org/10.14359/10649

Eid R, Kovler K, David I, Khoury W and Miller S (2018). "Behavior and Design of High-Strength Circular Reinforced Concrete Columns Subjected to Axial Compression". *Engineering Structures,* **173**:472-480. https://doi.org/10.1016/j.engstruct.2018.06.116

Han BS, Shin SW and Bahn BY (2003). "A Model of Confined Concrete in High-Strength Reinforced Concrete Tied Columns". *Magazine of Concrete Research*, **55**(3):203-214, https://doi.org/10.1680/macr.2003.55.3.203

Hong KN, Han SH and Yi ST (2006). "High-Strength Concrete Columns Confined by Low-Volumetric-Ratio Lateral Ties". *Engineering Structures*, **28**(9): 1346-1353. https://doi.org/10.1016/j.engstruct.2006.01.010

Hoshikuma J, Kawashima K, Nagaya K, and Taylor AW (1997). "Stress-Strain Model for Confined Reinforced Concrete in Bridge Piers". *Journal of Structural Engineering*, **123**(5):624-633. https://doi.org/10.1061/(ASCE)0733-9445(1997)123:5(624)

Hou C, Zheng W and Wu X (2020). "Structural State of Stress Analysis of Confined Concrete Based on the Normalized Generalized Strain Energy Density". *Journal of Building Engineering,* **31**:101321. https://doi.org/10.1016/j.jobe.2020.101321

Jaiswal DK, (2023). "Strength and Deformation of Solid and Hollow Confined RC Bridge Piers". PhD Thesis (*under preparation*), Department of Civil Engineering, Indian Institute of Technology Madras, Chennai, India

Kent DC, and Park R (1971). "Flexural Members with Confined Concrete". *Journal of Structural Engineering*, **97**(7): 1969-1990. https://doi.org/10.1061/JSDEAG.0002957

Lee JM, Kim YS, Kim SW, Park JH and Kim KH (2016). "Structural Performance of Rectangular Section Confined by Squared Spirals with No Longitudinal Bars Influencing the Confinement". *Archives of Civil and Mechanical Engineering,* **16**: 795-804. https://doi.org/10.1016/j.acme.2016.05.005

Mandal P (1993). "Curvature Ductility of RC Sections with and without Confinement". Masters Thesis, Department of Civil Engineering, Indian Institute of Technology Kanpur, India.

Mander JB, Priestley MJN, and Park R (1988). "Theoretical Stress-Strain Model for Confined Concrete". *Journal of Structural Engineering*, **114**(8):1804-1826. https://doi.org/10.1061/(ASCE)0733-9445(1988)114:8(1804)

Nagashima T, Sugano S, Kimura H and Ichikawa A (1992). "Monotonic Axial Compression Test on Ultra-High-Strength Concrete Tied Columns". *In Proceedings of the 10th World Conference on Earthquake Engineering*, July, 19-24pp

Park K, and Leslie PD (1977). "Curvature Ductility of Circular Reinforced Concrete Columns Confined by ACI Spiral". *6th Australasian Conference on the Mechanics of Stuctures and Materials*, Vol.1, 342-339pp.

Razvi S and Saatcioglu M (1999). "Confinement model for high-strength concrete". *Journal of Structural Engineering*, **125**(3): 281-289. https://doi.org/10.1061/(ASCE)0733-9445(1999)125:3(281)

Samra RM, Deeb NAA, and Madi UR (1996). "Transverse Steel Content in Spiral Concrete Columns Subject to Eccentric Loading". *ACI Structural Journal*, **93**(4): 412-419. https://doi.org/10.14359/9700

Scott BD, Park R, and Priestley MJN (1982). "Stress-Strain Behaviour of Concrete Confined by Overlapping Hoops at Low and High Strain Rates". *ACI Structural Journal*, **79**(1):13-27. https://doi.org/10.14359/10875

Sharma UK and Bhargava P (2005). "Behavior of confined high strength concrete columns under axial compression". *Journal of Advanced Concrete Technology*, **3**(2):267-281. https://doi.org/10.3151/jact.3.267

Sheikh SA, and Uzumeri SM (1982). "Analytical Model for Concrete Confinement in Tied Columns". *Journal of Structural Engineering*, **108**(12): 2703-2722. https://doi.org/10.1061/JSDEAG.0006100

Sheikh SA, and Yeh CC (1992). "Analytical Moment Curvature Relations for Tied Concrete Columns". *Journal of Structural Engineering*, **118**(2):529-544. https://doi.org/10.1061/(ASCE)0733-9445(1992)118:2(529)

Sheikh SA, and Toclucu MT (1993). “Reinforced Concrete Columns Confined by Circular Spirals and Hoops”. *ACI Structural Journal*, **90**(5): 542-553. https://doi.org/10.14359/3949

Shin HO, Yoon YS, Cook WD and Mitchell D (2015). “Effect of Confinement on the Axial Load Response of Ultrahigh-Strength Concrete Columns”. *Journal of Structural Engineering*, **141**(6):04014151. https://doi.org/10.1061/(ASCE)ST.1943-541X.0001106

Suzuki M, Akiyama M, Hong KN, Cameron ID and Wang WL (2004). “Stress-strain model of high-strength concrete confined by rectangular ties”. *In 13th World Conference on Earthquake Engineering*, August, Vol. 3330.

Wang W, Zhang M, Tang Y, Zhang X and Ding X (2017). “Behaviour of High-Strength Concrete Columns Confined by Spiral Reinforcement under Uniaxial Compression”. *Construction and Building Materials*, **154**: 496-503. https://doi.org/10.1061/(ASCE)ST.1943-541X.0001106

Wu T, Wei H, Zhang Y and Liu X (2018). “Axial Compressive Behavior of Lightweight Aggregate Concrete Columns Confined with Transverse Steel Reinforcement”. *Advances in Mechanical Engineering*, **10**(3): 1687814018766632. https://doi.org/10.1177/1687814018766632

Xie J, Elwi AE, and MacGregor JG (1997). “Performance of High Strength Concrete Tied Columns – A Parametric Study”. *ACI Structural Journal*, **94**(2):91-102. https://doi.org/10.14359/464

Yong YK, Nour MG, and Nawy EG (1988). "Behavior of Laterally Confined High-Strength Concrete under Axial Loads". *Journal of Structural Engineering*, **114**(2):332-351. https://doi.org/10.1061/(ASCE)0733-9445(1988)114:2(332)