A Review on Nonlinear Time History Analysis of Structures

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ABSTRACT

Nonlinear time history analysis (NTHA) is a powerful approach for assessing the seismic response of buildings, enabling engineers to better understand and predict the structural behaviour under dynamic loading. This review focuses on the tools, methods, and input parameters used in NTHA of buildings.

The review begins by introducing the concept of NTHA and its significance in evaluating the structural response to earthquakes. Various software tools commonly employed for conducting such analyses are discussed, including more common commercial programs and open-source alternatives often used by researchers. The capabilities, features, and limitations of these tools are mentioned to help researchers and practitioners choose the most suitable software for their specific needs. Additionally, the review covers strategies for modelling various structural components, including beams, columns, and connections, with an emphasis on accurately capturing the nonlinear behaviour.

Next, the review delves into the methods employed in NTHA. Some of the methods employed in NTHA are Direct integration method, Modal superposition technique and Fast nonlinear analysis (FNA) method.

The selection of input parameters plays a critical role in achieving reliable results in NTHA. This review discusses the key input parameters, such as damping models, ground motion records, and loading patterns.

In conclusion, this review provides insights into the tools, methods, and input parameters used in NTHA of buildings to help engineers and researchers in undertaking optimal seismic design and assessment of structures.

1 INTRODUCTION

Nonlinear Time History Analysis (NTHA) of buildings is a method that simulates the dynamic response of a structure under a given earthquake ground motion. Of all the seismic analysis methods, properly undertaken it is close to representing the reality of earthquake action on a structure. It is capable of accounting for the nonlinear behaviour of the structural elements, such as cracking, yielding, and failure, as well as the interaction between the structure and the foundation (Cruz et al., 1998), (Nguyen & Hjiaj, 2016), (Åldstedt & Bergan, 1978) and (Loli et al., 2014). NTHA is especially important for buildings that are irregular in geometry, mass, or stiffness distribution, or those with relatively complex structural systems, such as coupled walls, frames with infills, base isolations, and active or passive energy dissipators (Ko & Lee, 2009), (Vaiana et al., 2020), and (Prajwal et al., 2017). These buildings may exhibit nonlinear behavior at relatively low to moderate levels of ground motion, hence significant nonlinearity in their response to dynamic loads. Such nonlinearity may be a result of geometrical and/or material and/or structural systems/devices nonlinearity occurred within the structure and its elements. Therefore, NTHA is a necessary tool for the seismic analysis of buildings, especially for those that are complex, irregular, or have high performance requirements and structural systems.

Given NTHA is a technique to evaluate the dynamic response of structures under loading that varies over time, it requires solving a system of differential equations, often nonlinear, that describe the dynamic equilibrium and behaviour of the structure. The computational costs of NTHA are still considerable, compared with those of the static or modal analysis approaches and one of the challenges is to decrease the running time while not sacrificing the accuracy of the results (Tay & Chen, 2018) and (Bradley & Kantz, 2015).

There are a variety of the methods that are used to solve the equations of motion of the structure in NTHA. Most of these methods incorporate simplified assumptions and use numerical methods to solve the governing differential equations i.e. numerical integration methods to obtain numerical values of the functions and their derivatives and/or integrals. Some of the common such methods are Newmark-beta Method (Newmark, 1959), Wilson-Theta (Wilson et al., 1972), α-method of Hilber–Hughes–Taylor (HHT-Alpha), (Hilber et al., 1977) Central Difference method (Collatz 1966), Generalized Alpha (Chung & Hulbert, 1993), and the Trapezoidal Rule with the second order Backward Difference Formula (TR-BDF2) (Bank et al., 1985). The above-mentioned methods require dividing the whole-time interval of the problem into much smaller intervals to implement numerical techniques, and the small-time intervals are commonly known as time steps. These methods have been studied and compared from distinct aspects in (Zimmermann, 1987), (Broadbent, 1992), (Owren & Simonsen, 1995), and (Xie, 1996). Based on their results, some of the integration methods can provide accurate and stable solutions with reasonable computational cost and time step size, while some of them may distort the true response of the structure.

Moreover, to perform the NTHA, choosing an appropriate iterative algorithm is another crucial step. Iterative algorithms are used to solve the structure's dynamic equilibrium equations at each time step of the NTHA. Hence, the (Shamanskii 1967)) proposed a generalization of the Newton-Raphson and modified Newton algorithms, other studies led to Krylov-Newton (Scott & Fenves, 2010), Secant Newton (Crisfield, 1984), and the rank-two BFGS quasi-Newton procedure (Broyden 1970; Fletcher 1970; Goldfarb 1970; Shanno 1970).

Damping models are another crucial input parameters that represent energy dissipation in the structure, and significantly influence the accuracy and reliability of NTHA results. Given the damping of energy in a structure during vibration is influenced by several factors and sources such as sliding and/or yielding and/or cracking of the buildings structural and non-structural components as well as soil-structure interactions, it is practically impossible to accurately capture and model them all in a damping model, hence the simplified models and assumptions such as equivalent viscous damping assumption are often deployed for dynamic analysis. Some of the common damping models used in seismic analysis are Rayleigh (Rayleigh 1896), Caughey (Caughey 1960), Wilson-Penzien (Wilson and Penzien 1972), and Adhikari damping models (Adhikari (2000, 2006)). In common structural analysis software packages, Rayleigh damping model and in some packages the Wilson-Penzien damping model (modal damping) are typically used. Rayleigh damping is commonly used due to its simplicity and capability to represent structural energy dissipation. Modal damping, on the other hand, accounts for higher modes of vibration in a structure. Different studies have demonstrated that, some damping models may match the damping ratios for only a few modes of vibration and be unable to accurately represent the damping behaviour of the system and using classical viscous damping in nonlinear analysis can cause a variety of issues (Hall, 2006) and (Charney et al., 2016). A new approach should be developed that considers inherent damping as a nonlinear deformation-dependent and evolutionary path-dependent phenomenon, as suggested in (Carr 2007), (Puthanpurayil et al., 2011) and (Bowland et al., 2010). A better alternative is to use a damping model that gives a smooth curve with negligible oscillation and provides greater flexibility in matching damping ratios for a broad range of frequencies. To address those points the (Lee 2019), presented a new proportional damping model. It is particularly accurate in forming a constant modal damping ratio curve across a practical range of frequencies.

Moreover, considering hysteresis models for yielding elements in conducting the NTHA is crucial because they may adequately simulate the inelastic behavior of materials under cyclic loading, which occurs during seismic events. This includes models such as the Bouc-Wen, Takeda, or Clough-Penzien. These models are capable of representing the energy dissipation and pinching effects observed in structures during earthquakes.

The NTHA is a method to simulate the dynamic response of structures under seismic or other dynamic loads, considering the geometrical and material nonlinear behaviour of the structure. Some of the common software packages and tools to undertake NLTHA are ETABS and SAP2000 (both from Computers and Structures, Inc. (CSI)), Perform 3D, ABAQUS, Extreme Loading for Structures (ELS), RF-DYNAM PRO, OpenSees, program RUAUMOKO, ANSYS, COMSOL, and London University Stress Analysis System (LUSAS). Some case studies and comparison studies have assessed the accuracy of these software packages from specified criteria, such as results consistency (Wong 2013), software performance (Poon et al., 2011) and good agreement with experimental tests (Moldovan et al., 2014).

These studies collectively demonstrate the importance of software and tools in conducting the NTHA, and the need for further research and developments to enhance their capabilities.

2 METHODS EMPLOYED IN NTHA

Nonlinear time history analysis (NTHA) is a method to evaluate the dynamic response of structures under seismic loads, accounting for the nonlinearities resulted from geometry/deformations and/or structural systems/devices and/or materials behaviour. Numerical methods are often used to solve the governing differential equations that arise, based on dynamic equilibrium conditions, in the analysis of structures subjected to earthquake ground motions. The commonly used methods are Implicit and Explicit, depending on how the equations of motion are to be numerically solved. An implicit integration scheme is a numerical method used in time history analysis where the state of the system at a future time step is determined by solving a set of equations that include both the current and future states. The equilibrium equations at any time interval/step must be solved in implicit method. This means, for example, the global matrix form governing differential equations must be solved which imposes significant computational cost (i.e. inverting a stiffness matrix etc.). hence, this method is generally more stable and allows for larger time steps. In contrast, an explicit integration scheme calculates the future state of the system using only the current state information, which requires smaller time steps but is simpler and faster to compute. The equilibrium equations are not required to be solved at each time interval/step. While explicit methods are conditionally stable and often require a very small-time step, implicit algorithms are unconditionally stable and larger time steps can be used. The concept of ”conditional stability“ refers to the stability of a numerical method that depends on the parameters such as time step size. If the time step is not small enough in explicit method, the method may become unstable and produce inaccurate results The explicit methods are suitable for the analysis of short excitations, such as impulse excitation. The computational cost for the explicit approach does not vary as much as it does for the implicit technique; it is roughly proportionate to the size of the model (Sun et al., 2000). One instance of an explicit technique is the Central Difference Scheme.

On the other hand, an implicit technique is appropriate for dynamic analysis with comparatively long loading and response times, whether or not nonlinearities are present (Jia, 2014). The Newmark and Hilber-Hughes-Taylor (HHT) methods are the common implicit methods.

The integration scheme should provide accurate and stable solutions with reasonable computational cost and time step. In general, according to (Subbaraj and Dokainish 1989 a) and (Subbaraj and Dokainish 1989 b), implicit algorithms are effective for structural dynamic problems with a relatively small number of low-frequency modes. Explicit algorithms are more efficient for wave propagation problems where the contribution of intermediate and high-frequency modes to the response is crucial. Implicit algorithms are more prevalent in earthquake engineering problems due to computational efficiency. Moreover, in case of choosing implicit integration methods addressing a compatible iterative nonlinear solution algorithm is important. The iterative algorithm method should provide fast and robust convergence with reasonable computational cost and convergence criteria. The modified Newton, Krylov-Newton, and BFGS methods are some of the appropriate algorithm methods for NTHA.

2.1 Numerical Solutions for Nonlinear Problem

The equation of motions for a Multi degree of freedom (MDOF) system may be stated in matrix form as follows: (Jia, 2014).

$$[m]\{\ddot{x}\,(t)\} + [c]\{\dot{x}(t)\} + [k]\{x(t)\} = \{F(t)\} \tag{1}$$

Where $[k]$is the global stiffness, $[c]$ is Velocity dependent damping and $[m]$ is global mass matrices obtained from the assembly of the individual element matrices. $\{x(t)\}$, $\{\dot{x}(t)\}$,and $\{\ddot{x}\,(t)\}$ representing the displacement, velocity and acceleration vectors of the system respectively. and $\{F(t)\}$ denotes the force vector.

Exact/analytical/closed form solutions to equations of motions are usually not possible for large, geometrically complicated, and multi element systems/structures. This becomes even more challenging when the system is nonlinear and/or the excitation is random (such as an earthquake record), hence difficult or impossible to be represented other than approximately/numerically. Several methods can be adopted to find such solutions in an approximate manner, such as direct integration, free oscillation, Duffing’s equation etc. (Thomson, W.1966), (Hashemi & Ahmadi, 2016) and (Hashemi, G. 2023). The direct integration method is widely used for both analytical and computer-based analysis due to its accuracy, but it is also computationally expensive and complex (Gavrea et al., 2005). Most of the structures are modelled as multi degree of freedom (MDOF) systems with two or more modes of vibration. The modal superposition technique transforms the equations of motion of the structure from the physical coordinate system into a modal coordinate system. The modal superposition method and the direct integration method are two distinct techniques utilized for solving problems in dynamic analysis. Linear dynamic analysis problems are frequently solved using modal superposition technique, as the responses of a structure/system are proportionate to the loads/excitations applied to it. However, the modal superposition technique is not suitable for nonlinear problems since it assumes that the structure's response is a linear combination of its natural modes of vibration.

The Fast nonlinear analysis (FNA) (Wilson, E.L. 2002) is an approach that breaks down nonlinearities into lumped elements and uses Ritz vectors instead of eigen vectors. (Li et al., 2021), for details see (Wilson, E.L. 2002).and (Akar & Willner, 2019). FNA is potentially a useful approach for designers of new buildings, such as base isolation, or where capacity design is used, provided it is implemented carefully where appropriate e.g. by specifying and guaranteeing the yielding elements' locations. However, the FNA, while being fast, is not accurate for all structures but only for those structures in which the number of nonlinearity-prone elements compared with total number of elements is relatively small (Puthanpurayil, and Sharpe 2022). Such an example is a stacked modular system with non-linear dampers between each module. A more accurate technique which does not have this FNA's limitation is numerical time-history direct integration (DI) non-linear dynamic analysis, although, as mentioned before, it has a high computational cost (Newmark, 1959).

To sum up, the choice of the best method depends on the trade-off between accuracy, stability, efficiency, as well as the specific features of the problem.

3 INPUT PARAMETERS

Choosing the appropriate input parameters for nonlinear time history analysis of structures are important, as they influence the accuracy and reliability of the structural response. However, some of the parameters may have more influence than others, depending on the problem characteristics, analysis objectives, and performance criteria. For example, the selection and scaling of ground motion records are crucial in achieving reliable results in nonlinear time history analysis (Málaga et.al., 2008).

The structural model, which includes structural elements and connections, is another important parameter which should be created carefully and calibrated and validated based on experimental data or design codes. For instance, the fiber-beam element model has been widely used in the elasto-plastic analysis of earthquake engineering, and its accuracy has been verified for many times (Qin 2005). and has high solution efficiency and accuracy when used for predicting the response of the whole concrete structures under earthquake. (Spacone et al.,1996) and (D'Ambrisi & Filippou, 1999). It is widely known (e.g., (Petrini et al., 2008)) that the choosing an appropriate damping model to simulate the energy dissipation in structures subjected to dynamic loading, such as earthquakes, play a crucial role in achieving reliable results in NTHA. The Rayleigh damping model, a widely used and simple form of damping model, assumes a linear combination of the mass and stiffness matrices for the damping matrix. To improve the Rayleigh model, the Caughey damping model was developed, which uses a polynomial function of the natural frequency to match damping ratios for multiple modes. In addition to address the challenges of above-mentioned damping models, the Wilson-Penzien model matches damping ratios for each mode through the solution of a generalized eigenvalue problem.

Adhikari's research (Adhikari 2000) and (Adhikari 2006) has led to the development of generalized proportional models that use basis functions to extend the Rayleigh model. Recently, Lee (Lee 2019) proposed a new proportional damping model that can match a damping ratio curve in the frequency domain.

It should be noted that different damping models may have varying advantages and disadvantages for different types of engineering problems. Therefore, it is crucial to choose the appropriate damping model for NLTHA of buildings and to verify the sensitivity, compatibility, and reliability of the results.

4 TOOLS FOR NTHA

Nonlinear Time History Analysis (NTHA) of buildings is a crucial aspect of research and practice in the field of civil engineering. Consequently, a variety of software tools have been developed to facilitate NTHA. To select the most appropriate software for specific needs, it is important to consider the capabilities, features, and limitations of these tools. Among the most used software packages for NTHA are ETABS, SAP2000, Perform3D, OpenSees, ABAQUS, ELS, RF/DYNAM Pro, and LUSAS. The distinctions between these software packages lie in their element library, material models, load types, analysis options, solver strategy, output formats, and user interface. However, comprehensive information about each software package may be obtained from their corresponding developers and available guidelines. Researchers, and practitioners in the field of civil engineering need to carefully evaluate the array of software tools available for NTHA and select the most appropriate software that satisfies their specific needs.

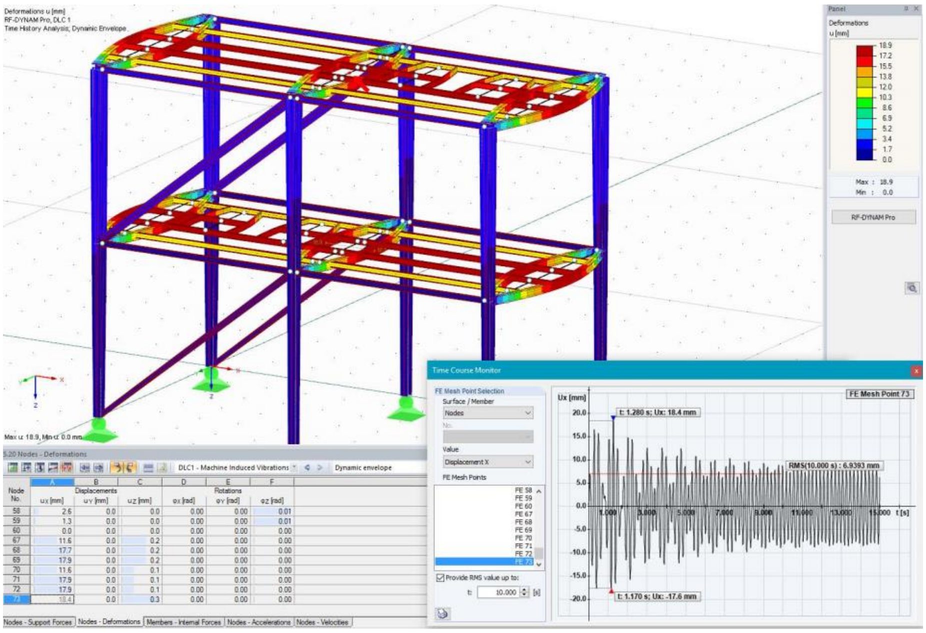

Figure 1: A view of some windows and options in the RF-/DYNAM Pro

It is important to note that ETABS, SAP 2000, and PERFORM 3D are all commercial software products created by Computers and Structures, Inc. (CSI). These programs mostly use lumped plasticity and distributed plasticity approach for capturing nonlinear behaviour of beam and columns and utilize panel zone approach for connections. (Computers and Structures, Inc. - Technical Knowledge Base. (n.d.)). A classification of concentrated and distributed plasticity models may be found in (Reinhorn et al., 2010).

The RF- /DYNAM Pro - Nonlinear Time History Add-on Module for RFEM/RSTAB is a commercial add-on module for RFEM and RSTAB, which are structural analysis software for planar and spatial structures. This module allows for nonlinear dynamic analysis to external excitation and benefit from Fiber-based approach for modelling nonlinear behaviour of beams and columns. (DYNAM Pro-Nonlinear Time History, 2023)

Extreme Loading for Structures (ELS) is another commercial structural analysis software that is founded on the applied element method (AEM). In this software Fiber-based method and snap option (software feature to align, attach, or intersect objects with other objects or grid points using different snap modes) apply to capture nonlinear behaviour of structural elements and connections respectively. (Structural Engineering Software Program & Nonlinear Structural Analysis, 2023) and (Extreme Loading® for Structures. Theoretical Manual 2023).

Among the most accurate commercial software for finite element analysis is ABAQUS, which can model diverse types of materials and perform different analyses. ABAQUS is a Finite Element Method (FEM) program and not one typically used for NTHA of complete structural systems. In this software package, the

Fiber-based elements and sharing nodes approach (for connections) employ to capture the nonlinearity in models. (Abaqus/Standard, Abaqus, Inc., Providence, RI, 2024).

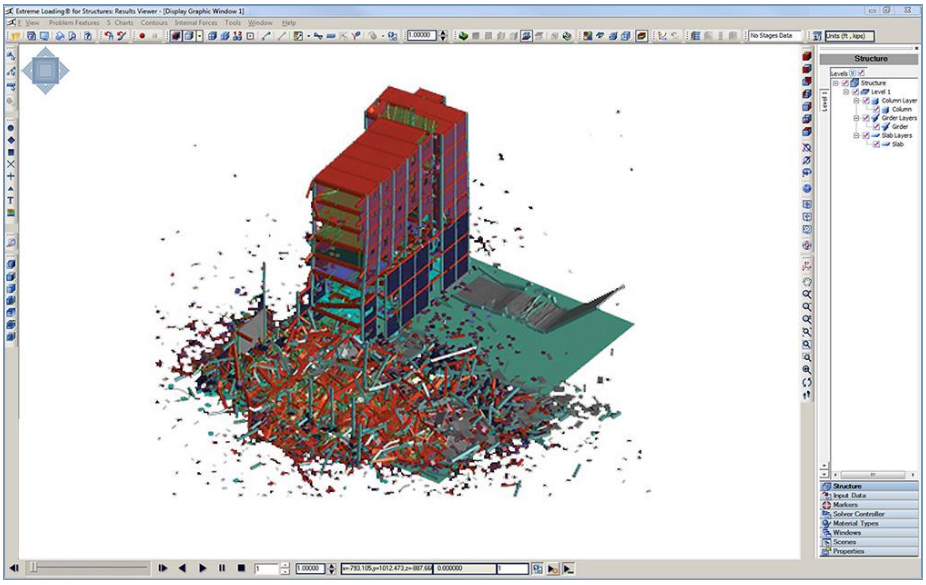

Figure 2: A display graphic window and options in the ELS software

The OpenSees software framework is an open-source tool that can simulate the seismic response of structural and geotechnical systems. It is a powerful tool for analysing different structures that has good reputation in research community. In this software a wide range of methods including Fiber-based approach and rotational spring approach are included to consider the nonlinearity in elements and connections respectively. (Open System for Earthquake Engineering Simulation - Home Page. (n.d.).)

On the other hand, LUSAS Civil & Structural is a commercial finite element analysis software application that offers comprehensive facilities for analysing, designing, and assessing all types of structures - above and below ground. This software also uses a wide variety of strategies such as Geometric and material nonlinear approach and gap approach, to reflect the nonlinearity in elements and connections respectively. (LUSAS Programmable Interface (LPI) Customisation and Automation Guide).

ANSYS is another commercial software package that offers various engineering simulation tools for different physical fields. One of these tools is ANSYS structural analysis software, which can perform finite element analysis (FEA) on different types of structures. With this software, users can simulate transient nonlinear phenomena, modal analysis, and linear static analysis. (ANSYS 2023). The software also supports various material models, element types, and solver options. It can handle complex models with multiple parts and interactions.

The general-purpose simulation programme COMSOL is another numerical analysis tool commonly used by researchers and industry (Multiphysics, C. 1998). COMSOL software has fully linked Multiphysics and single-physics modelling capabilities, a comprehensive modelling process from geometry to results assessment, and tools for creating and integrating simulation programmes. (COMSOL. Structural Mechanics Module, User's Guide).

Both ANSYS and COMSOL are Multiphysics simulation tools that offer a wide range of capabilities and applications, including nonlinear time history analysis of buildings (Achkar et al., 2008). ANSYS has been found to be effective for finite element analysis of different structures, including masonry structures, with key considerations including structural modeling, material constitution, and iterative algorithms (Li-ping, 2006). COMSOL, on the other hand, is known for its capability in modeling and simulation for engineers and scientists, with a focus on practical applications and examples (Tabatabaian, 2014). Both tools have been compared in terms of simulation time and quality, with COMSOL found to be faster while maintaining the same level of precision (Achkar, 2008).

RUAUMOKO (Carr 2003), the Māori god of earthquakes, volcanoes, and seasons, is name of a software package for nonlinear time history analysis (NLTHA) of buildings, developed by Professor Athol Carr from the University of Canterbary. Ruaumoko 2D is a finite element method-based package designed for the analysis of the response of structures to earthquake excitation. The program is designed to run in both interactive and batch modes, with the ability to read input data from various file formats (Carr 2005). The program prompts for responses to a series of questions and can plot accelerograms or excitation histories scaled to the magnitudes used in the analysis. The 3D version of software is also developed (Carr 2015). RUAUMOKO 3D utilizes a local scratch file for random access, performing a read and write operation for each structure member at every iteration and time-step. On the other hand, Using a local file for analysis can slow down Ruaumoko 3D when run over a network and work folder is on a server (Carr 2015). RUAUMOKO offers several options for modelling the Mass, Damping, and Stiffness matrices for a structure. RUAUMOKO also offers the widest range of hysteresis models for non-linear elements of the structure available from any of the NTHA programs. The stiffness representation has many different member types available, including frame, spring, and truss members, among others. Ruaumoko uses the explicit or implicit integration methods to solve the Dynamic Time-history analysis. According to (Carr 2017), the software uses Newmark constant average acceleration and Central Difference explicit method.

SPACE GASS is a commercial multi-purpose 3D analysis and design program for structural engineers. Its extensive range of features makes it suitable for modelling and analysing structures ranging from beams, trusses and frames to buildings, towers, tanks, cable structures and bridges. Capabilities include a 64-bit multi-core solver, 3D rendered graphics, plate finite elements, frame elements, cable elements, tension/compression-only elements, moving loads and links to many CAD and building management programs. The results may also be exported to several CAD and BIM programmes. To speed up simulations, SPACE GASS also makes use of an intelligent matrix solver that runs on several cores. Structure elastic behaviour is modelled by SPACE GASS using the well-established stiffness approach in conjunction with Wavefront and Paradise equation solvers. However, the Space GaSS cannot undertake nonlinear time-history analysis and its non-linear static analysis solver only employs the Newton-Raphson approach.

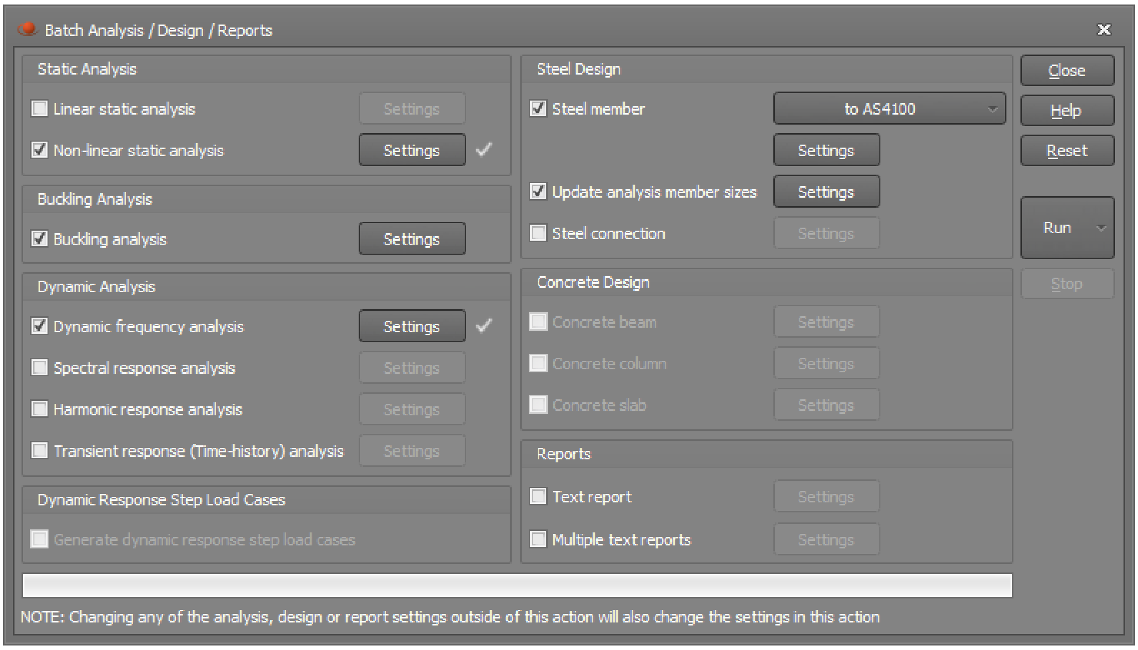

Figure 3: Different Analysis Options window in SPACE GASS software

In general, commercial software packages provide a wide range of built-in features and have a more user-friendly interface than open-source software packages. However, commercial software packages are often expensive and require a powerful computer to run efficiently. Open-source software packages are free and provide a wide range of features, but they have a steep learning curve and require some programming knowledge to use effectively making them more suitable for research community. Software packages such as ETABS CSI, SAP2000 CSI, and Perform 3D are commonly regarded as having easy-to-learn user-friendly graphical interfaces in conducting nonlinear Time History Analysis of buildings. The software tool should also offer a wide range of integration methods, iterative algorithm methods, and damping models, and avoid unrealistic assumptions in the functioning. (Chopra, 2012). Hence, ABAQUS (mostly in component scale), OpenSees, RUAUMOKO and LUSAS are some of the appropriate software tools for conducting NTHA.

5 CONCLUSION

This review provides a general information and insights into the typical tools, methods, and input parameters used in NTHA of buildings. Accurate selection of ground motion records, reliable structural modelling, accurate material properties, appropriate nonlinear analysis settings, reliable algorithms, and convergence criteria are essential factors for an accurate dynamic analysis. It is crucial that such parameters and strategies reflect, as accurately as possible, the realistic behaviour and characteristics of the structures being modelled and analysed.

It is important to note that the guidelines provided in this paper are general recommendations and may not cover all the requirements or considerations of specific projects. It is essential to adapt and fine-tune these guidelines based on the unique characteristics of each project. The input parameters are better, depending on the available resources, be obtained, as much as possible, from material tests, and if not possible, from the design codes and/or well-established empirical formulas. Any required simplifications, approximations, or assumptions need to be made carefully and rationally.

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